Why It is No Longer a Good Idea to Be in The Investment Industry

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A spurious tail is the performance of a certain number of operators that is entirely caused by luck, what is called the "lucky fool" in Taleb (2001). Because of winner-take-all-effects (from globalization), spurious performance increases with time and explodes under fat tails in alarming proportions. An operator starting today, no matter his skill level, and ability to predict prices, will be outcompeted by the spurious tail. This paper shows the effect of powerlaw distributions on such spurious tail. The paradox is that increase in sample size *magnifies* the role of luck.

The Spurious Tail

The idea is well known (see Taleb 2001), that as a population of operators in a profession marked by a high degrees of randomness increases, the number of stellar results, and stellar for completely random reasons, gets larger. The "spurious tail" is therefore the number of persons who rise to the top for no reasons other than mere luck, with subsequent rationalizations, analyses, explanations, and attributions. The performance in the "spurious tail" is only a matter of number of participants, the base population of those who tried. Assuming a symmetric market, if one has for base population 1 million persons with zero skills and ability to predict starting Year 1, there should be 500K spurious winners Year 2, 250K Year 3, 125K Year 4, etc. One can easily see that the size of the winning population in, say, Year 10 depends on the size of the base population Year 1; doubling the initial population would double the straight winners. Injecting skills in the form of better-than-random abilities to predict does not change the story by much.

Because of scalability, the top, say 300, managers get the bulk of the allocations, with the lion's share going to the top 30. So it is obvious that the winner-take-all effect causes distortions: say there are N initial participants and the "top" M managers selected, the result will be M/N managers in play. Let us set the tail probability=M/N and derive K the threshold level that would be expected to arise just from randomness. As the base population gets larger, that is, N increases linearly, K increases in a convex manner as we push into the tail probabilities.

This is quite paradoxical as we are accustomed to the opposite effect, namely that a large increases in sample size reduces the effect of sampling error; here the narrowness of M puts sampling error on steroids.

Aggravation Under Fat Tails: The issue is acute if one limits the experiments to the binomial distribution as we started doing in the thought experiment at the beginning of the introduction, with K already quite impressive. Now if we introduce fat tails, the level of K for a given M/N becomes shockingly large.

While much has been done to extract randomness from performance, what this note does is show with the simplest possible thought experiment the size ok K under fat tailed regimes, and how much more severe the problem becomes.

The distribution in the tails

Assume the following dynamics of the returns x, with mean 0, with f the Student T Distribution with α is the tail exponent

$$f(x) = \frac{\left(\frac{\alpha}{\alpha + x^2}\right)^{\frac{\alpha}{2}}}{\sqrt{\alpha} \operatorname{B}\left(\frac{\alpha}{2}, \frac{1}{2}\right)}$$

where the Beta Distribution B(a, b) = $\Gamma(a) \Gamma(b) / \Gamma(a+b) = \int_{0}^{1} t^{a-1} (1-t)^{b-1} dt$.

Let us use the mean absolute deviation (in place of standard deviation in common use in the conventional literature)

$$E[|x|] \frac{2\sqrt{\alpha}}{(\alpha-1) \operatorname{B}\left(\frac{\alpha}{2}, \frac{1}{2}\right)}$$

The probability of exceeding K normalized in mean deviation terms is, with $\alpha > 1$,

$$P_{>K} = \frac{\alpha + (K - \alpha K) {}_{2}F_{1} \left(\frac{1}{2}, \frac{\alpha + 1}{2}; \frac{3}{2}; -\frac{K^{2} (\alpha - 1)^{2} B \left(\frac{\alpha}{2}, \frac{1}{2}\right)^{2}}{4 \alpha^{2}}\right)}{2 \alpha}$$

Where $_2F_1$ is the Hypergeometric $_2F_1(a, b; c; z) = \sum_{k=0}^{\infty} (a)_k (b)_k / (c)_k z^k / k!$

We have N participants; M is the number of "winners" to whom funds are allocated, ranked as the maximum M returns $\{x_1, \ldots, x_M\}$ from n returns over the period concerned. As N grows and M does not grow (or to the least does not grow at the same rate), we end up getting higher and higher K deviations.

Setting the tail probabilities, with *N* variable and *M* constant, we have the lower bound of the spurious tail $K_{\alpha}(M)$ such as $\frac{M}{N} = P_{>K}$, we derive $K_{\alpha}(M)$ numerically for different values of α in Figure 1 and the table below (actually, for large values of K_{α} , using the asymptotic distribution of power laws, it can be simplified to $C\left(\frac{M}{N}\right)^{-1/\alpha}$, where C is a constant, with its sensitivity to α quite significant, $\partial K/\partial \alpha = C \frac{\left(\frac{M}{N}\right)^{-1/\alpha} Log\left[\frac{M}{N}\right]}{\alpha^2}$, as

power laws, it can be simplified to C $\left(\frac{1}{N}\right)$, where C is a constant, with its sensitivity to α quite significant, $\partial K/\partial \alpha = C \frac{\alpha^2}{\alpha^2}$, as can be seen in Figure 1 for $\Delta \alpha = 1/2$).

The econophysics literature estimates α between 5/2 and 3 (the "cubic" exponent) for the general stock market, though individual stocks have been shown to be even more thick-tailed.



Figure 1: For M=30, the size of the spurious tail, K in mean deviation, as N the number of track records increase. Two cases, a = 2, borderline infinite variance; 5/2, moderately fat-tailed, but with finite variance. We can see K reaching levels of 120 mean deviations.

This exercise is a thought experiment stripped to the bare bones. However, the simplification makes the point milder than it would be under a richer model.

Note 1: Homogeneity. we are assuming an entirely homogeneous population of operators, and scaling by absolute deviations. Injecting variations in the population would accentuate the effect, particularly if some operators have a higher variance than others.

Note 2: Path Independence (absence of absorbing barrier). If one adds filters of maximum excursions on the path (maximum drawdown,etc.), then the problem worsens further: the point is not to just have a certain level performance, but a certain performance *conditional of not having hit* an absorbing barrier. (We skip the math here to make the point even simpler).

Note 3: Normalizing Returns. By scaling to mean deviations we avoid the ills of the coefficient of variation based on the $\mathcal{L}2$ Norm, "Sharpe ratio" which is uninformative under power laws.

Comparison to Thin Tailed Distributions

Table 1 allows the comparison to a situation with Gaussian returns (that is, Lognormal prices). for $N = 10^6$, K is between 68 and 178 mean deviations compared to 5 for a Gaussian.

 Table 1: The "spurious tail" for different values of N, expressed in units of MAD, Mean Absolute Deviations at different levels of α tail exponents compared to the Gaussian used here as control. Assume M=30.

N	Probability	$K_{\alpha=5/2}(M)$	$K_{\alpha=2}(M)$	K(M),Gaussian
1000.	0.03	4.	5.46218	2.35723
2000.	0.015	6.	7.89868	2.7198
5000.	0.006	8.	12.6366	3.14851
100000.	0.0003	28.	56.5885	4.30089
500000.	0.00006	52.	126.077	4.8204
$1. imes 10^6$	0.00003	68.	178.001	5.02931

Conclusion and Generalization

The "fooled by randomness" effect grows under connectivity where everything on the planet flows to the "top x", where x is becoming a smaller and smaller share of the top participants. Today, it is vastly more acute than in 2001, at the time of publication of (Taleb 2001). But what makes the problem more severe than anticipated, and causes it to grow even faster, is the effect of fat tails. For a population composed of 1 million track records, fat tails multiply the threshold of spurious returns by between 15 and 30 times.

Genralization: This condition affects any business in which prevail (1) some degree of fat-tailed randomness, and (2) winner-take-all effects in allocation.

To conclude, if you are starting a career, move away from investment management and performance related lotteries as you will be competing with a swelling future spurious tail. Pick a less commoditized business or a niche where there is a small number of direct competitors. Or, if you stay in trading, become a market-maker.

References

Taleb, Nassim N. (2001, 2004), Fooled by Randomness, Random House and Penguin.